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Effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers

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ABSTRACT

An analysis is presented to investigate the effects of chemical reaction, thermal stratification, Soret number and Dufour number on MHD free convective heat and mass transfer of a viscous, incompressible and electrically conducting fluid on a vertical stretching surface embedded in a saturated porous medium. The governing partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically using a fourth order Runge-Kutta scheme with the shooting method. The results obtained show that the flow field is influenced appreciably by the presence of chemical reaction, thermal stratification Soret number, Dufour number and magnetic field.

1. Introduction

The range of free convective flows that can occur in nature and in engineering practice is very large and has been extensively considered by Jaluria [1]. Erickson et al. [2] studied the problem of heat and mass transfer in the laminar boundary layer flow of moving flat surface with constant surface velocity and temperature, focusing on the effect of suction/injection. The problem of heat and mass transfer on a stretching sheet with suction or blowing was investigated by Gupta and Gupta [3]. And coupled heat and mass transfer by natural convection in a fluid saturated porous medium has many important applications in geothermal and geophysical engineering such as the extraction of geothermal energy, the migration of moisture in fibrous insulation, underground disposal of nuclear waste, and the spreading of chemical pollutants in saturated soil. In addition, the heat and mass transfers simultaneously affecting each other that will cause the cross-diffusion effect. The mass transfer caused by the temperature gradient is called Soret effect, while the heat transfer caused by the concentration gradient is called Dufour effect. Eckert and Drake [4] presented several cases of Dufour effect. Weaver and Viskanta [5] have pointed out that when the differences of the temperature and the concentration are large or when the difference of the molecular mass of the two elements in a binary mixture is great, the coupled interaction is significant. A primary discussion on the effect of the cross-coupled diffusion in a system with horizontal temperature and concentration a gradient was made by Malashew and Gaikad [6].

Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Dursunkaya and Worek [7], Anghel and Takhar [8], Postelnicu [9] are worth mentioning. Very recently, Alam and Rahman [10] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. Yih [11] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction/blowing, whereas Watanabe [12] investigated the behaviour of the boundary layer over a wedge with suction injection in forced flow. Anjali Devi and Kandasamy [13] studied effects of chemical reaction, heat and mass transfer on non –linear MHD laminar boundary layer flow over a wedge with suction and injection.

Recently Postelnicu [14] study influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. In our work we study effects of chemical reaction, Soret number, Dufour

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number and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media.

2. Mathematical analysis

Two-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, electrically conducting in the presence of a uniform magnetic field has been considered with heat, mass transfer, chemical reaction and thermal stratification in porous medium considering Soret and Dufour effects. According to the coordinate system, the x-axis is chosen parallel to the vertical surface and the y-axis is taken normal to it. A transverse magnetic field of strength B_0 is applied parallel to the y-axis. The fluid properties are assumed to be constant and the chemical reaction is taking place in the flow. The physical properties ρ, μ, D and rate of chemical reaction k_c are constant throughout the fluid. Under the boundary layer assumptions, the governing equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \left(\frac{\nu}{k_1} + \frac{\sigma B_0^2}{\rho}\right)u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_P}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + \upsilon\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} - k_c(C - C_\infty), \tag{4}$$

The boundary conditions for this problem can be written as

$$\begin{aligned} &u = U(x), \quad \upsilon = 0, \quad T = T_w(x), \quad C = C_w(x) \quad \text{at } y = 0 \\ &u = 0, \quad T = T_\infty(x) = (1-n)T_0 + nT_w(x), \quad C = C_\infty \quad \text{as } y \to \infty \end{aligned} \tag{5}$$

where n is a constant which is the thermal stratification parameter and is such that $0 \le n < 1$. The n defined as thermal stratification parameter is equal to $m_1/(1+m_1)$ of Nakayama and Koyama [15] where m_1 is a constant. B_0, σ and T are the strength of magnetic field, the electrical conductivity of the fluid and the temperature. k_c is the rate of chemical reaction, ρ is the density, g is the acceleration due to gravity, β_T is coefficient of volume expansion, β_C is the volumetric coefficient of expansion with concentration. ν, k_1 and D_m are kinematics viscosity, permeability of porous media and coefficient of mass diffusivity respectively, C_P is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio and C_s is the concentration susceptibility.

The following transformation can be introduced:

$$\psi = (\nu x U(x))^{1/2} f(\eta), \qquad \eta = (U(x)/\nu x)^{1/2} y, \tag{6}$$

this choice as Acharya et al. [16] and we introduce that:

$$U(x) = ax, \tag{7}$$

$$\theta(\eta, x) = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w - T_\infty = d(x) = sx^n, \tag{8}$$

$$\phi(\eta, x) = \frac{C - C_\infty}{C_w - C_\infty}, \quad C_w - C_\infty = d_1(x) = s_1 x^{n_1}, \tag{9}$$

It can be easily verified that the continuity Eq. (1) is identically satisfied. And here a is dimensional constant, the values of s, s_1 and n_1 are constants. Then introducing the relations (6)–(9) into Eqs. (2)–(4) with considering T_∞ is constant, we obtain the following dimensionless ordinary differential equations.

$$f''' + Gr_x Re_x \theta + Gc_x Re_x \phi - \left(M^2 Re_x + \frac{Re_x}{K}\right)f' - f'^2 + ff'' = 0, \tag{10}$$

$$\theta'' + Pr(Du\phi'' - nf'\theta + f\theta') = 0, \tag{11}$$

$$\phi'' + Sc(Sr\theta'' - \gamma Re_x \phi - n_1 f'\phi + f\phi') = 0, \tag{12}$$

where primes denote differentiation with respect to η and $Re_x = Ux/\nu$ is the Reynolds number, $Gr_x = \nu g\beta_T(T_w - T_\infty)/U^3$ is the Grashof number, $Gc_x = \nu g\beta_C(C_w - C_\infty)/U^3$ is the modified Grashof number, $Pr = (\mu C_P)/k$ is the Prandtl number, $Sc = \nu/D_m$ is the Schmidt number, $M^2 = (\sigma B_0^2 \nu)/\rho U^2$ is the magnetic parameter, $K = (k_1 U^2)/\nu^2$ is the permeability parameter, $\gamma = (\nu k_c)/U^2$ is the chemical reaction parameter, $Du = (D_m k_T/\nu C_s C_P)(C_w - C_\infty/T_w - T_\infty)$ is the Dufour number, $Sr = (D_m k_T/\nu T_m)(T_w - T_\infty/C_w - C_\infty)$ is the Soret number.

The corresponding boundary conditions are given by

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \tag{13a}$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{13b}$$

The above system (10)–(12) with the boundary conditions (13) have been solved numerically for various values of parameters by applying Runge-Kutta method. Also the quantities of physical interest in this problem are the local Nusselt and Sherwood numbers, which are defined by

$$Nu = \frac{-x}{T_w - T_\infty}\left.\frac{\partial T}{\partial y}\right|_{y=0} = -(Re_x)^{1/2}\theta'(0) \quad \text{then}$$

$$\frac{Nu}{(Re_x)^{1/2}} = -\theta'(0), \tag{14}$$

$$Sh = \frac{-x}{C_w - C_\infty}\left.\frac{\partial C}{\partial y}\right|_{y=0} = -(Re_x)^{1/2}\phi'(0) \quad \text{then}$$

$$\frac{Sh}{(Re_x)^{1/2}} = -\phi'(0), \tag{15}$$

and from that we can introduce numerical results of the local Nusselt number and the local Sherwood number as well as the velocity, temperature and concentration profiles are presented for different physical parameters and at $\gamma = 1$, $Re_x = 1$, $Sc = 0.62$, $n = 0.5$, $M^2 = 1$ and $K = 1$.

Sr	*Du*	*Nu*	*Sh*
2	0.03	0.67079	0.77313
1	0.12	0.83502	0.82108
0.5	0.3	0.91135	0.93950
0.1	0.6	1.00521	1.16126

3. Results and discussion

Numerical calculations have been carried out for different values of M, Du, Sr, γ, n, Sc, Pr and for fixed values of $Gr_x = 1$, $Gc_x = 2$, $Re_x = 1$, $K = 1$ and $n_1 = 0.5$ to obtain a clear insight of the physical problem, numerical results are displayed with the help of graphical illustration. And numerical results are summarized in Tables 1–6, the velocity profiles for different values of Soret and Dufour numbers are presented in Fig. 1. It is observed that the velocity of fluid decreases as Soret number decreases and Dufour number increases. Fig. 2 represents the temperature profiles for different values of Soret and Dufour numbers and constant chemical reaction and thermal stratifications parameters. It is observed that the temperature of fluid decreases with the decrease of Soret number and an increase of Dufour number. Fig. 3 represents the concentration

Table 1
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\phi'(\infty)$ for various values of Sr with Du at $M^2=1$, $\mathrm{Re}_x=1$, $\gamma=1$, $\mathrm{Pr}=0.71$, $\mathrm{Sc}=0.62$, $n=0.5$, $Gr_x=1$, $Gc_x=2$ and $K=1$

Sr	Du	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(0)$	$-\theta'(\infty)$	$-\phi'(\infty)$
2	0.03	0.53993	0.77313	0.67079	0.11646	0.07953	0.17000
1	0.12	0.59861	0.82108	0.83502	0.09802	0.07267	0.11493
0.5	0.3	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776
0.1	0.6	0.69801	1.16126	1.00521	0.06804	0.00931	0.06570

Table 2
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\phi'(\infty)$ for various values of γ with $M^2=1$, $Re_x=1$, $Pr=0.71$, $Sc=0.62$, $Gr_x=1$, $Gc_x=2$, $n=0.5$, $K=1$, $Sr=0.5$ and $Du=0.3$

γ	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(\infty)$	$-\theta'(\infty)$	$-\phi'(\infty)$
0.3	0.57160	0.87710	0.67940	0.10012	0.04590	0.12637
0.5	0.59349	0.89609	0.75115	0.09507	0.04704	0.11337
0.7	0.61335	0.91408	0.81819	0.09058	0.04809	0.10207
1.0	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776

Table 3
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\phi'(\infty)$ for various values of M^2 with $\gamma=1$, $Re_x=1$, $Pr=0.71$, $Sc=0.62$, $Gr_x=1$, $Gc_x=2$, $n=0.5$, $K=1$, $Sr=0.5$ and $Du=0.3$

M^2	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(\infty)$	$-\theta'(\infty)$	$-\phi'(\infty)$
1	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776
2	0.98278	0.90425	0.89921	0.07234	0.06331	0.09599
3	1.28137	0.87542	0.88965	0.06291	0.07560	0.10249

Table 4
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\gamma'(\infty)$and $\phi'(\infty)$ for various values of n with $\gamma=1$, $Re_x=1$, $Pr=0.71$, $Sc=0.62$, $Gr_x=1$, $Gc_x=2$, $M^2=1$, $K=1$, $Sr=0.5$ and $Du=0.3$

n	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(\infty)$	$-\theta'(\infty)$	$-\phi'(\infty)$
0.2	0.63470	0.85322	0.93416	0.08608	0.05637	0.08783
0.5	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776
0.9	0.64628	1.04692	0.88276	0.08314	0.04159	0.08768

Table 5
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\phi'(\infty)$ for various values of Sc with $\gamma=1$, $Re_x=1$, $Pr=0.71$, $n=0.5$, $Gr_x=1$, $Gc_x=2$, $M^2=1$, $K=1$, $Sr=0.5$ and $Du=0.3$

Sc	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(\infty)$	$-\theta'(\infty)$	$-\phi'(\infty)$
0.22	0.55728	1.02080	0.57932	0.11037	0.00284	0.19025
0.42	0.60391	0.97544	0.75809	0.09445	0.03017	0.12465
0.62	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776
0.82	0.66909	0.90981	1.04735	0.07868	0.06404	0.06618

Table 6
Results of $f''(0)$, $\theta'(0)$, $\phi'(0)$, $f''(\infty)$, $\theta'(\infty)$ and $\phi'(\infty)$ for various values of Pr with $\gamma=1$, $\mathrm{Re}_x=1$, $Sc=0.62$, $n=0.5$, $Gr_x=1$, $Gc_x=2$, $M^2=1$, $K=1$, $Sr=0.5$ and $Du=0.3$

Pr	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(\infty)$	$-\theta'(\infty)$	$-\phi'(\infty)$
0.3	0.60810	0.61081	0.99149	0.09932	0.16865	0.07475
0.5	0.62470	0.77779	0.95102	0.09137	0.09964	0.08294
0.71	0.63997	0.93950	0.91135	0.08472	0.04952	0.08776
0.9	0.65216	1.07505	0.87774	0.07989	0.01789	0.08999

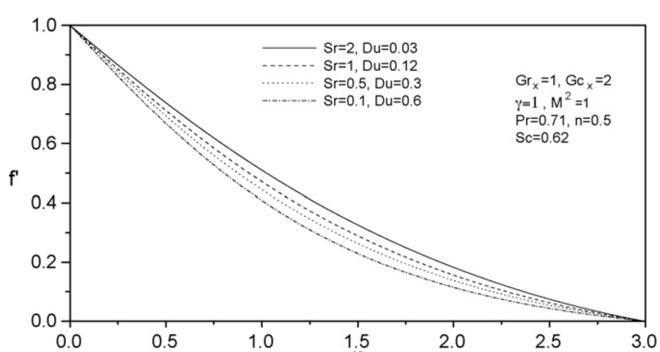

Fig. 1. Velocity profiles for different Soret and Dufour numbers.

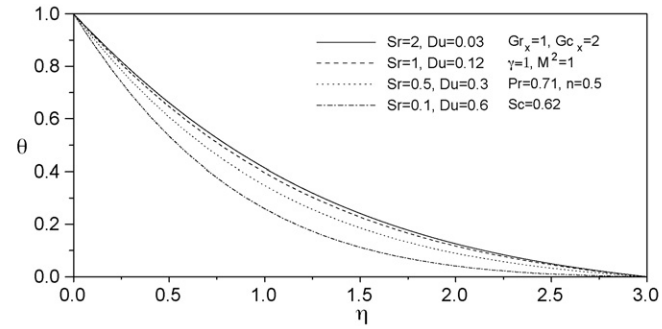

Fig. 2. Soret with Dufour effect over the temperature profiles.

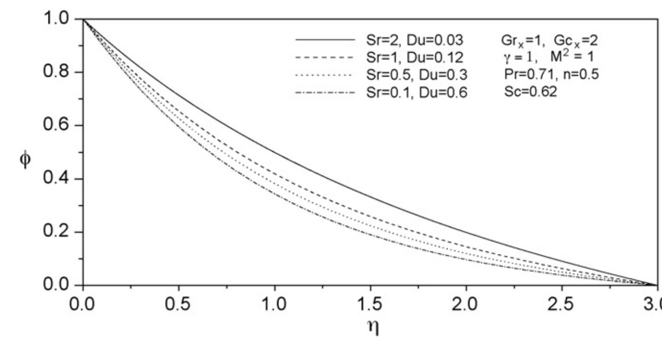

Fig. 3. The effect of Soret and Dufour over the concentration profiles.

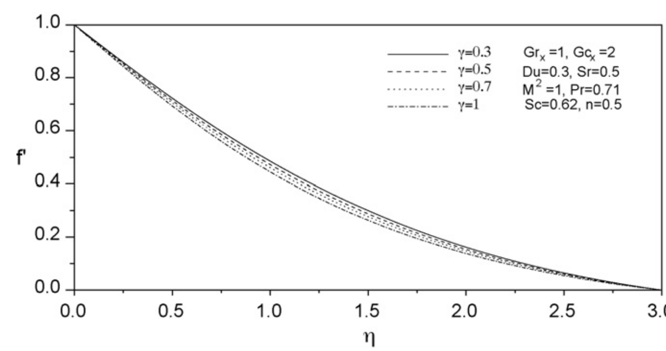

Fig. 4. Chemical reaction effect over the velocity profiles.

profiles for different values of Soret and Dufour numbers and constant chemical reaction and thermal stratifications parameters. It is observed that the concentration of fluid decreases with decreasing Soret number and increasing Dufour number. Figs. 4–6 represent the effect of chemical reaction on velocity, temperature and con-

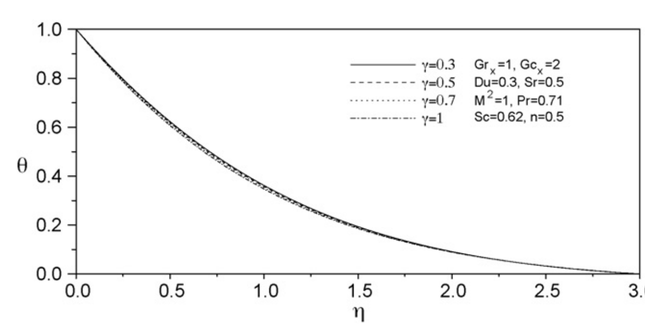

Fig. 5. Chemical reaction effect over the temperature profiles.

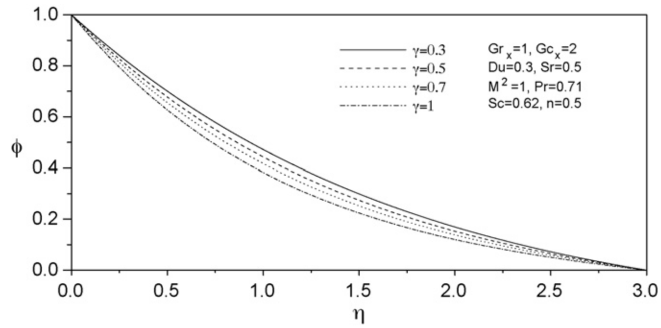

Fig. 6. The effect of chemical reaction over the concentration profiles.

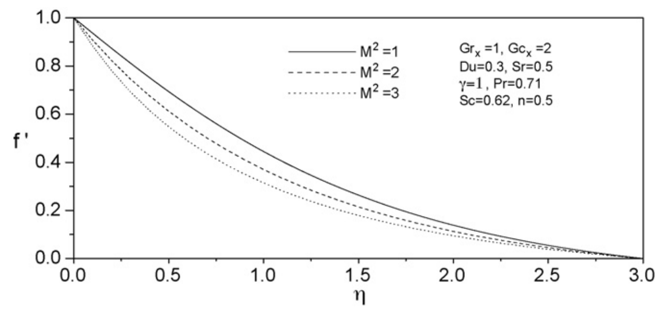

Fig. 7. Magnetic field effect over the velocity profiles.

centration functions. It is observed that for increasing chemical reaction the velocity and concentration profiles decreases and a small change for temperature profiles. Figs. 7–9 represent the effect of magnetic parameter on velocity, temperature and concentration functions. It is observed that the velocity of the fluid decreases with the increase of magnetic parameter, the value of temperature profiles are increase with the increase of magnetic parameter and the concentration of the fluid have a small increase with the

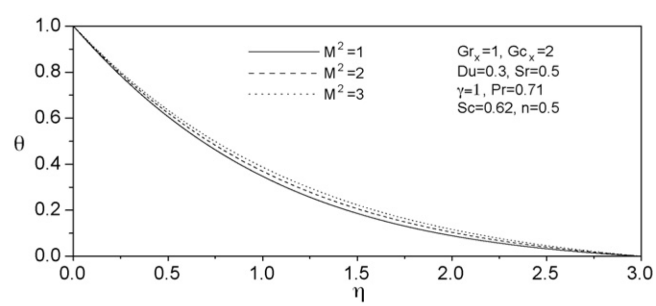

Fig. 8. Magnetic field effect over the temperature profiles.

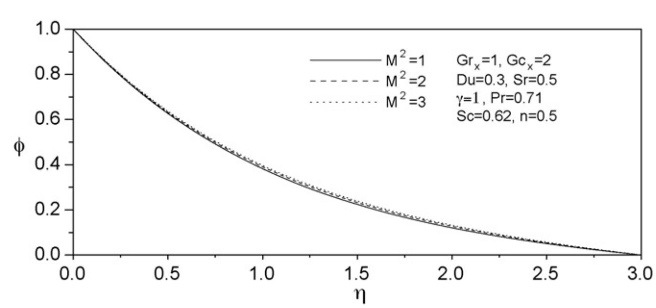

Fig. 9. Magnetic field effect over the concentration profiles.

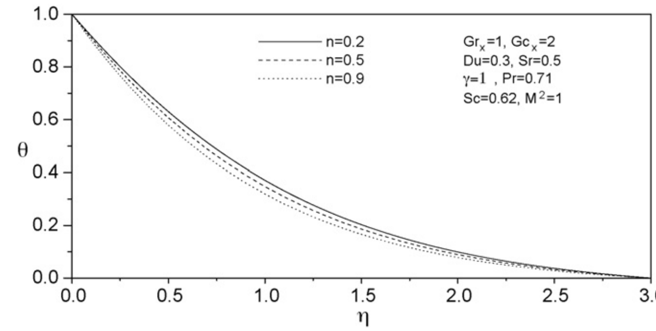

Fig. 10. The effects of thermal stratification over the temperature profiles.

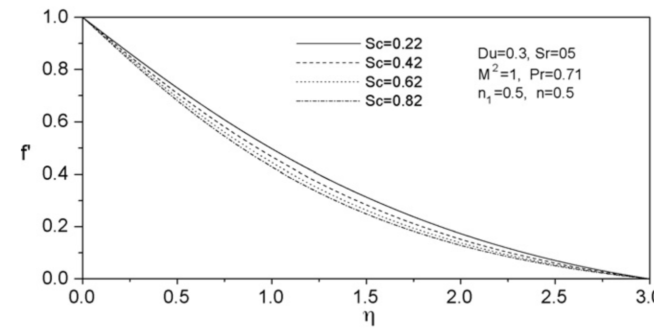

Fig. 11. The effect of Schmidt number over the velocity profiles.

increase of the magnetic parameter. Fig. 10 shows the effect of thermal stratification over the temperature profiles. It is observed that the temperature of the fluid decreases with the increase of the thermal stratification parameter. Fig. 11 represents the effect of Schmidt number over the velocity profiles. It is seen that the velocity of the fluid decreases with an increase of the Schmidt number. Fig. 12 represents the effect of Schmidt number on the temperature profiles. It is observed that the temperature of fluid increases with an increase of the Schmidt number. Fig. 13 demonstrates the dimensionless concentration profiles for different values of Schmidt number with constant chemical reaction, magnetic parameter, Soret number and Dufour number. It is observed that the concentration of the fluid decreases with an increase of the Schmidt number. Figs. 14–16 demonstrate the effect of Prandtl number over the velocity, temperature and concentration profiles. From Figs. 14 and 15 it is seen that the velocity and temperature profiles are decrease for the increase of the Prandtl number. From Fig. 16 we can observe that the concentration of the fluid increases with the increase of Prandtl number. Figs. 17–19 demonstrate the effect of Dufour

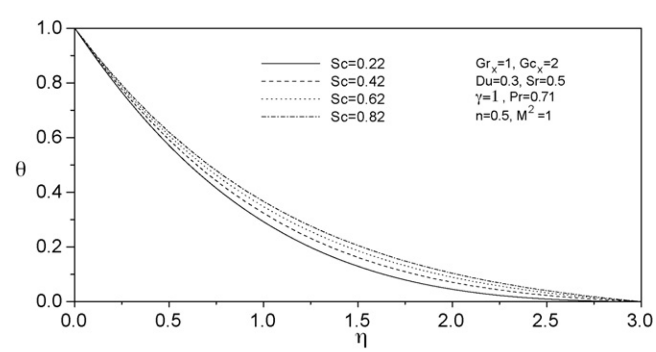

Fig. 12. The effect of Schmidt number over the temperature profiles.

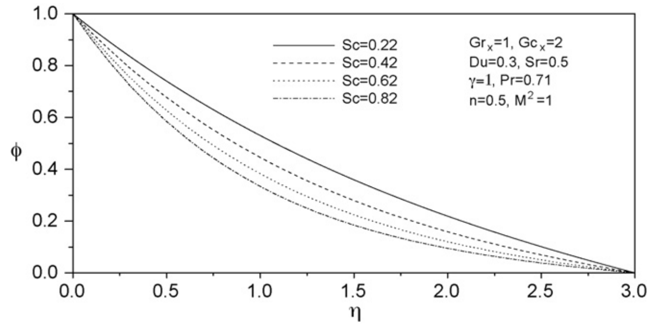

Fig. 13. The effect of Schmidt number over the concentration profiles.

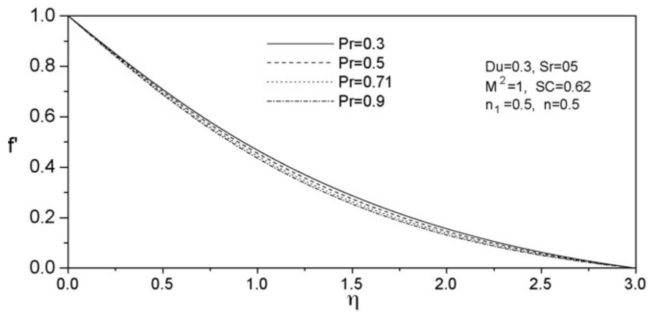

Fig. 14. Prandtl number effect over the velocity profiles.

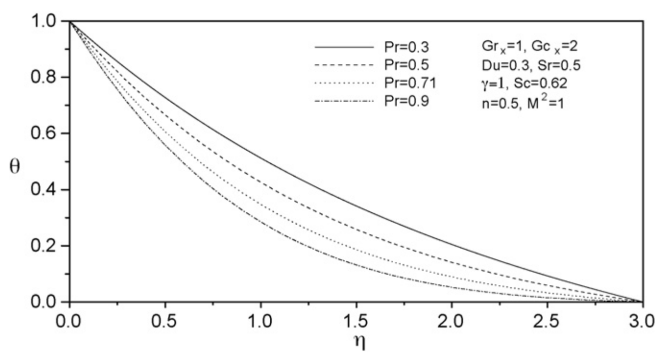

Fig. 15. Prandtl number effect over the temperature profiles.

number on the velocity, temperature and concentration profiles, respectively, it is seen that with increase of Dufour number the velocity profiles and concentration profiles are decrease. The concentration profiles are increase with the increase of the Dufour number.

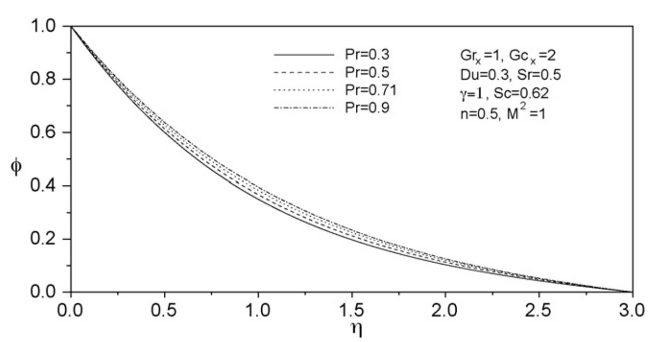

Fig. 16. Prandtl number effect over the concentration profiles.

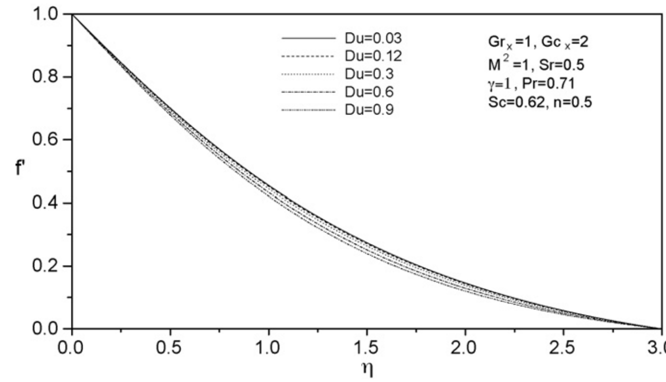

Fig. 17. Dufour number effect on the velocity profiles.

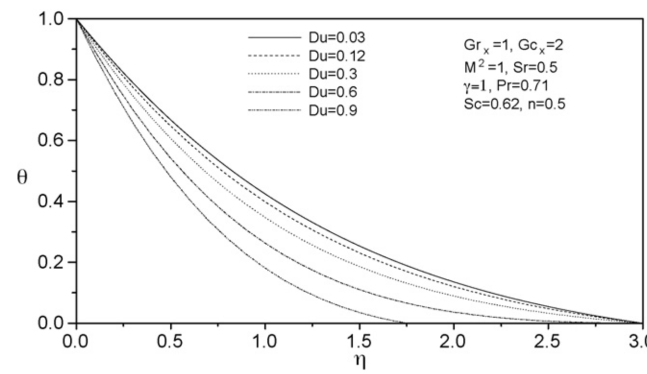

Fig. 18. Dufour number effect on the temperature profiles.

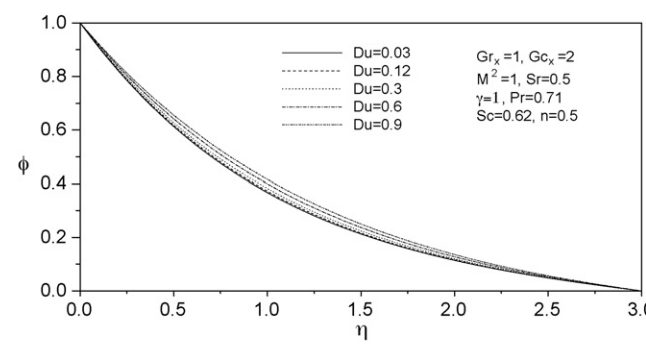

Fig. 19. Dufour number effect on the concentration profiles.

4. Conclusion

In this paper, we have studied numerically the effects of the chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers. Numerical solutions for the governing equations for momentum, energy and concentration are given. Tabulated values and graphical representations are presented for the velocity, thermal and concentration functions for various values of Soret number with Dufour number, chemical reaction parameter, magnetic field parameter, thermal stratification parameter, Schmidt number, Prandtl number and Dufour number.

- Due to Soret number with Dufour number, chemical reaction parameter, magnetic field parameter, Schmidt number, Prandtl number and Dufour number, the velocity of the fluid decrease with the increase of these parameters. And the velocity profiles decrease with the decrease of Soret number.
- The temperature profiles are decrease with the increase of Dufour number with the decrease of Soret number, thermal stratification parameter, Prandtl number and Dufour number. It has a small decrease with the increase of chemical reaction. And the temperature profiles are increase with the effects of magnetic field parameter and Schmidt number.
- The concentration profiles are decreased with the increase of Dufour number with the decrease of Soret number, chemical reaction and Schmidt number. It has a small increase with the increase of magnetic field parameter. And the concentration profiles are increased with the effects of Dufour number and Prandtl number.

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